

# Evolved Flocking Under Predation in Simple Vehicles

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## Abstract

Increasing evidence points to predation as a likely source of evolutionary pressure behind flocking behaviours in animals. In simulation, flocking is shown to be a highly stable strategy for simple agents with steering behaviours, under (co-) evolutionary pressure of predation. A number of perturbations are undergone to assess the stability of the behaviour.

**Keywords:** flocking; schooling; herding; group behaviour; evolution; locomotion; predation.

## Introduction

Collective behaviours have long held our fascination - from the flocking of birds to the complexities of human social interactions. Even such entities as bacteria and simple physical processes have been shown to exhibit characteristics of emergent group dynamics (Vicsek et al., 1995; Schauder & Bassler, 2001). The relative advantages of coordinated group behaviour in flocks, schools and herds of animals have been widely debated. Do such behaviours arise as a response to predation, foraging or reproductive needs? What abilities must a species have for flocking to develop? How do purely individual needs give rise to such well-coordinated collective traits?

Group or swarm behaviours may be understood as a form of collective decision-making, in which multiple agents pool their sensory and bodily resources in order to perform cognitive tasks to the benefit of the group (Visscher & Camazine, 1999). Couzin (2009) raises a number of questions for further research on collective behaviours in animals, relevant to the study of cognition: “How important is complexity at the level of individuals to collective animal behavior?”; “To what extent can effective collective decision-making result from competitive interactions among selfish individuals?”. Evolutionary simulation methods may offer a useful angle from which to assess the answers to such questions.

A simulation is presented in which flocking is shown to be a highly robust and likely strategy - as a result of predation - in minimally-cognitive agents capable of directly perceiving relevant aspects of each other’s motion. The aim is to form a simple framework in which to assess the evolutionary stability of flocking behaviours. From this beginning, light may be shed on factors stimulating or inhibiting flocking in more complex group-behaviour settings.

## Background

### The Biology of Flocking

Several studies support a strong link between flocking and predation in bird species. Flocking has been observed to be

reduced under reduced predation (Beauchamp, 2004), to increase individual survival rates, and to reduce foraging efficiency in redshanks (Cresswell, 1994), as well as akepa (Hart & Freed, 2005).

Other work has measured positions and velocities of individuals within flocks - giving precise analyses of the dynamics within fish shoals (Tien et al., 2004), and flocks of starlings (Ballerini et al., 2008). Importantly for current purposes, both studies confirm a strong link between biological flocking and the simulation methods described below.

### Simulated Flocking

Reynolds (1987) proposed a model for flocking behaviours based on local rules - as well as a system for simulating realistic autonomous agents using steering forces (1999). Reynolds’ system for flocking agents, known as Boids, has proved successful - and is widely used as a model for real-world flocking in academia and (entertainment) industry. Little advance has been made on Reynolds’ three simple rules for flocking since their inception:

1. Collision avoidance: avoid collisions with nearby flockmates
2. Velocity matching: attempt to match velocity with nearby flockmates
3. Flock centering: attempt to stay close to nearby flockmates (Reynolds, 1987)

These are commonly referred to as the rules of separation, alignment and cohesion. In recent years, advances have been made in the mathematics (Toner & Tu, 1998) and algorithmic efficiency (Olfati-Saber, 2006) of calculating Boid-like flocking. Increasingly, studies have combined simulated flocking with simulated evolution, as well as computational neuroscience - to explore flocking as an adaptive and cognitive phenomenon.

### Evolving Intelligent Flocking

Cliff and Miller (1994) give an account of the challenges inherent in neural coevolution of pursuit and evasion, and present a successful simulation - postulating the emergence of flocking given a greater simulation scale. Reynolds (1996) presents a simulation for coevolving the game of tag using Genetic Programming, demonstrating improvements in pursuit and avoidance strategies in steering agents.

More recently, agents have been successfully developed to evolve flocking given neural control systems. (Ward et al., 2001) successfully coevolved flocking in simulated fish with neural networks, as a response to predation. This useful result

demonstrates the possibility of investigating coevolving coordinated group behaviours in simulation, as a cognitive phenomenon. Kwasnicka et al. (2007) describe a model in which (neural) coevolution of flocking is attempted under predation - with success. The authors admit the evolution required some help to achieve flocking. Another study coevolves boids already able to flock, using a model in which flocking is observed to increase as a direct response to predation, rather than to foraging (Wood & Ackland, 2007). Recent and ongoing work looks into complex neuro-evolution of specialisms in group behaviour (Nitschke et al., 2007). The above approaches, aided by improvements in computing power, have advanced our understanding of flocking as an evolutionary phenomenon. There is progress still to be made in providing a clear characterisation of the ecological niche occupied by flocking behaviours, and of the cognitive and physiological attributes necessary for agents to develop them.

### Method

A simulation is created in which two-dimensional mobile agents - predators and prey - coevolve. The predators are rewarded for their hunting ability with reproductive success. When prey die (from fixed-length old age, predation or fast collisions with obstacles), a new prey agent is created with a genome mutated from one of the surviving prey. The relative likelihood of a prey agent being selected to reproduce in this way increases linearly as its lifespan continues.

Predators and prey slow down in collisions with each other and with obstacles, and have weighted connections mapping sensors to steering. These weights constitute the agents' genomes, which evolve via a haploid process. The aim of the simulation is to ascertain whether, given the exact sensory information known to be sufficient to produce realistic flocking, such behaviours will be favoured over other possible strategies - and if so, how sensitive such behaviour will be to perturbation. Note that the sensory information useful for flocking (average position and heading of nearby fellow prey) is also sufficient for a range of other behaviours, e.g. avoidance, clustering, collision, dispersal.

To model the agents and environment, Reynold's OpenSteer engine for C++ was used (2004). The code for the simulation is available as a "plugin" for OpenSteer (Sayers, 2009).

### Vehicles

Following the method of Reynolds (1999), for each prey agent a steering force  $O_{prey}^{\vec{}}(t)$  is applied in each time step (constrained by a maximum force  $F_{max}$ ). This steering force, divided by the mass  $m$ , gives the agent's acceleration -  $a_{prey}^{\vec{}}(t)$  - which, added to the current velocity  $v_{prey}^{\vec{}}(t)$ , and constrained by a maximum speed  $v_{max}$  (as well as a maximum deviation angle  $\alpha_{max}$  from the agent's forward direction), gives the resultant position  $x_{prey}^{\vec{}}(t)$  at each time step  $t$ .

### Sensors

Figure 1 shows the regions of the prey vehicle's perception. Following (Aoki, 1982; Reynolds, 1987; Wood & Ackland,

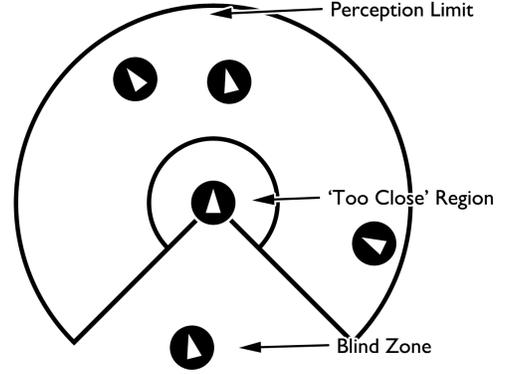


Figure 1: Prey perception areas.

Table 1: Prey sensors.

Sensor	Type	Value
$S_{prey}^1$	Fellow prey position	$\frac{1}{n} \sum_{i=1}^n x_{prey_i}^{\vec{}}$
$S_{prey}^2$	Fellow prey velocity	$\frac{1}{n} \sum_{i=1}^n v_{prey_i}^{\vec{}}$
$S_{prey}^3$	Fellow prey "too close"	$\frac{1}{n} \sum_{i=1}^n \left( \frac{r_{close}}{ x_{prey_i}^{\vec{}} } - 1 \right) x_{prey_i}^{\vec{}}$
$S_{prey}^4$	Obstacle	$\frac{1}{m} \sum_{i=1}^m \left( \frac{r_{max}}{ x_{obst_i}^{\vec{}} } - 1 \right) x_{obst_i}^{\vec{}}$
$S_{prey}^5$	Predator	$\frac{1}{p} \sum_{i=1}^p \left( \frac{r_{max}}{ x_{pred_i}^{\vec{}} } - 1 \right) x_{pred_i}^{\vec{}}$

2007), there is an inner ( $r_{close}$ ) and outer ( $r_{max}$ ) area of perception, as well as a maximum angle of perception. When active, the (vector) values given by the prey sensors are shown in Table 1, where  $x_{prey_i}^{\vec{}}$  are the relative positions of perceived fellow prey for  $i = 1$  to  $n$ ,  $v_{prey_i}^{\vec{}}$  is their relative velocity,  $x_{obst_i}^{\vec{}}$  are the mean nearest points of ( $m$ ) perceived obstacles, and  $x_{pred_i}^{\vec{}}$  are the mean relative locations of ( $p$ ) nearby predators. All sensors are active within  $r_{max}$  except  $S_{prey}^3$ , which is only active within  $r_{close}$ . When inactive, sensors return zero-vector values.

Note that for the sensors  $S_{prey}^3$ ,  $S_{prey}^4$  and  $S_{prey}^5$ , the resultant vectors are inverted in length so that their magnitudes increase as the target becomes closer. This corresponds to the intuitive sense that these sensors are likely to be more useful to prey agents in this form - for developing the capacities of avoidance of predators, obstacles and fellow prey members. Prey may evolve to converge on or diverge from fellow prey in two ways, due to the presence of sensor  $S_{prey}^1$ .

We give the vehicles idealised sensory and locomotive abilities, known to be capable of producing flocking (as well as other behaviours) when combined with the given control system, in order to assess the evolutionary stability of any group behaviours that may ensue. Potential further work lies in in-

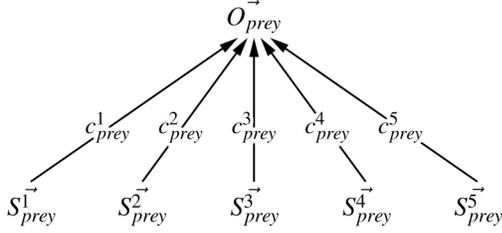


Figure 2: Prey control network.

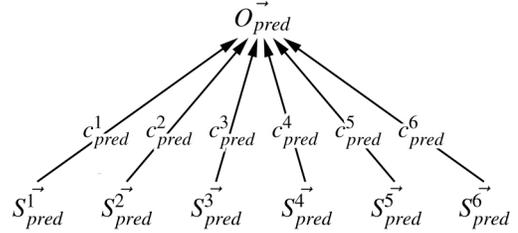


Figure 3: Predator control network.

investigating how the use of more physically realistic sensors (e.g. in the style of Beer’s (2000) work on minimally cognitive behaviour) might affect the evolutionary stability of the observed behaviours.

Table 2: Predator sensors.

Sensor	Type	Value
$S_{pred}^1$	Target prey position	$\vec{x}_{preyT}$
$S_{pred}^2$	Target prey velocity	$\vec{v}_{preyT}$
$S_{pred}^3$	Obstacle	$\frac{1}{m} \sum_{i=1}^m \left( \frac{r_{max}}{ \vec{x}_{obst} } - 1 \right) \hat{x}_{obst}$
$S_{pred}^4$	Fellow predator position	$\frac{1}{p} \sum_{i=1}^p \vec{x}_{pred'}$
$S_{pred}^5$	Fellow predator velocity	$\frac{1}{p} \sum_{i=1}^p \vec{v}_{pred'}$
$S_{pred}^6$	Fellow predator “too close”	$\frac{1}{p} \sum_{i=1}^p \left( \frac{r_{close}}{ \vec{x}_{pred'} } - 1 \right) \hat{x}_{pred'}$

The predators’ sensors (Table 2) and control systems (Figure 3) are similar to the prey’s, with the addition of sensors detecting target prey ( $prey_T$ ) position ( $S_{pred}^1$ ) and velocity ( $S_{pred}^2$ ). A predator chooses a target by searching for the nearest prey - considering slow-moving prey as priority over faster moving nearer prey. This factor was added to the simulation during development (giving predators more realistic behaviours and punishing prey “clumping”), and is found to increase the speed of, but be non-essential in the favouring of flocking behaviour in prey. The predators’ vision is not angle-limited as the prey’s is, and predators are able to pick out prey from an arbitrary distance.

Each agent has a genome containing connection weights ( $c_{prey}^{1...5}$ ,  $c_{pred}^{1...6}$ ) from sensors to output used in the following linear control system equations (corresponding to Figures 2 and 3 and Tables 1 and 2):

$$\vec{O}_{prey} = \frac{1}{5} \sum_{i=1}^5 c_{prey}^i S_{prey}^i$$

$$\vec{O}_{pred} = \frac{1}{6} \sum_{i=1}^6 c_{pred}^i S_{pred}^i$$

## Interactions

As well as sensing their environment and each other, vehicles are faced with physical interactions. They can bump into each other, or into obstacles. When they collide into each other, their maximum speed is reduced to a minimum ( $v_{min}$ ). Thereafter it increases linearly, returning to its previous value. Similarly with obstacles, a collision causes slowing. When a predator meets a prey, it “eats” it - the prey’s body is consumed (and another prey is regenerated by mutating one of it’s fellows). When eating prey, predators slow down to  $v_{min}$ , and recover, in the same way as for collisions.

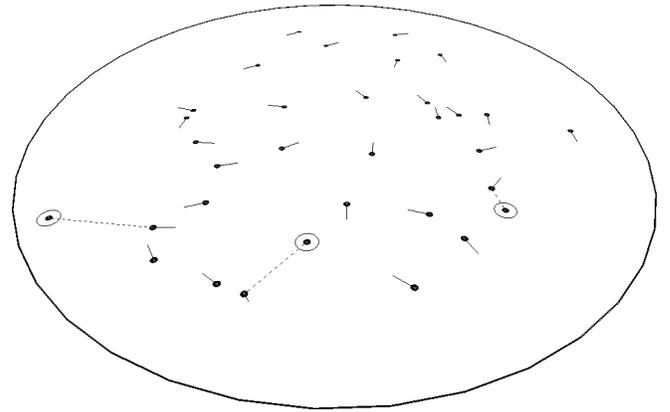


Figure 4: A typical run - start. Predators are ringed. Short lines show velocities. Dashed lines denote shortest paths from predators to target prey

## Results

Firstly, we analyse results for a model version of the simulation in which 30 prey and 3 predators coevolve, using averaged data from 30 runs. Stable, realistic flocking develops in 87% of cases in the given time period. We then consider the results of various perturbations of the system in order to assess the limits of this stability.

## Model Runs

A view of the simulation environment can be seen in Figure 4. This image is taken at the start of a run, when the prey have weights set at normally distributed random values with mean zero. The predators and prey in a majority of runs coevolve

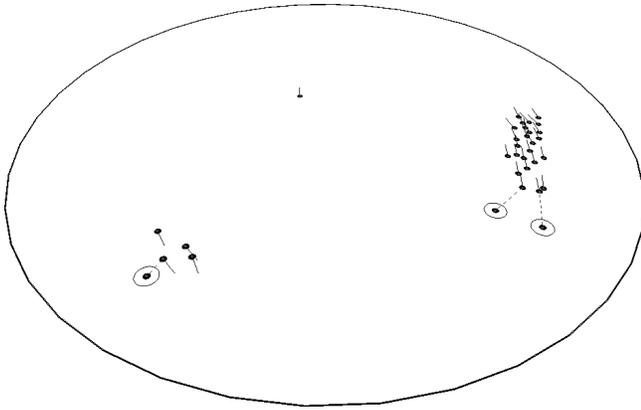


Figure 5: A typical run - later. Flocking has developed.

to give a scene as in Figure 5 - a stable state of flocking behaviour.

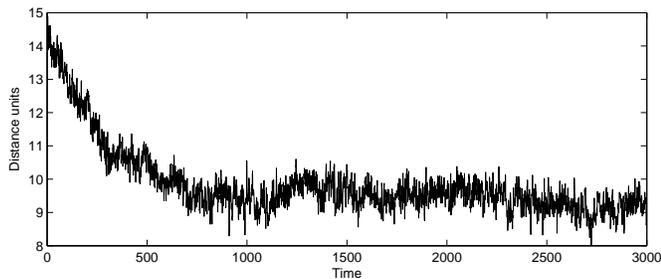


Figure 6: Inter-prey nearest neighbour distance - average of 30 runs

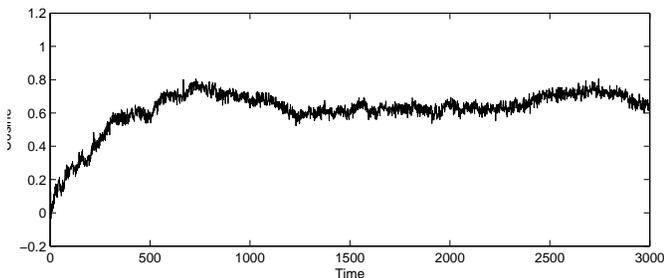


Figure 7: Inter-prey cosine of angular deviation - average of 30 runs

In Figures 6 and 7 we see the development of the inter-prey nearest neighbour distance and angle - clearly showing a tendency towards a smaller mean distance (cohesion and separation) and greater cosine of angular deviation (alignment). The averaged data is sampled once every 500 frames of animation. Approximately 200 prey and 40 predators are regenerated every 100 of the resultant time units (taking around 5 minutes in a time frame making the motion appear realistic), as shown on these and following figures.

The predators evolved in all observed runs to use a positive linear combination of the sensors  $S_{pred}^1$  and  $S_{pred}^2$  - giving a predictive pursuit strategy. The predators do not evolve strong obstacle (perimeter) avoidance - presumably due to pursuit and the topology of the environment. The predators do not form any identifiable group behaviours within the population size used.

### Perturbations

Given the successful coevolution of flocking with predation described above, we perturb the system to assess the stability of this arrangement.

**Population sizes** Experimenting with the ratio of predators to prey, whilst keeping the combined population roughly equal, reveals that flocking is a remarkably robust strategy in this regard: the prey continue to reliably evolve flocking when they are equal in number to predators (20 of each). Pushing the ratio much further than this in the predators' favour, flocking continues to develop until a point is reached where prey do not survive long enough to find each other, and thus are unable to develop group behaviours. Altering the ratio in the opposite direction, flocking evolves even with many prey and just one predator - see Figure 8.

With no predators present, the prey do not evolve flocking, but rather tend to collide with one another, forming tight slow-moving clumps.



Figure 8: 100 prey, 1 predator (detail).

In highly constrained runs, where the perimeter radius is small compared to the number of prey, flocking does not evolve. Rather a misalignment-cohesion behaviour develops where the prey hit into each other repeatedly in tight clusters.

Flocking does not occur below a certain threshold number of prey - around twelve for the perimeter size used in the "Model Runs" section. This should be expected - the prey regenerate to random positions in the environment when eaten, so low numbers do not allow time for flocking to occur.

Overall the tendency towards flocking behaviour under predation is observed to be resilient to variations in population density and ratio.

**Physical constraints** In the simulation runs described above, the major external physical constraints on the predators and prey occur in the actions of bumping and eating. Both slow the agents down - their speed gradually increasing afterward. If the predators are not slowed down by eating prey, flocking does *not* evolve - rather, a dispersive, mutually-

avoiding behaviour emerges. This suggests that the emergence of flocking is dependent on the flock being able to get away in the time taken for the predator to deal with its catch (or “handling time”). Experimenting with gradually reintroducing this delay reveals what may be a phase transition, in which the likelihood of flocking greatly increases above a threshold handling time.

Agents are internally constrained by their maximum speed and force parameters. In the simulations described above, the predators have greater values for these parameters, allowing them to outrun the prey. Flocking still evolves (albeit at a greatly reduced rate) when the predators’ maximum force and speed are set to slightly lower values than the prey’s. Thus, as in nature, the predators are able to capitalize on mistakes and weaknesses (collisions) and survive on flocks of creatures normally able to outrun them. Below a (gradual) threshold, the predators are unable to kill enough prey for co-evolution to occur.

**Eliminating sensors** We may reduce the abilities of predators and prey, and still achieve flocking. Of the inter-prey sensors, removal of sensor  $S_{prey}^1$  (nearby prey average position) results in similar flocking patterns to when it is present. It might be argued that cohesion in this case is enforced by the presence of a circular boundary. This suggests that cohesion is a longer-range effect than alignment or separation.

On removing the sensor  $S_{prey}^2$  (nearby prey average velocity) the prey evolve mostly mutually avoidant behaviour with high drift. On removing sensor  $S_{prey}^3$  (“too close” prey average position), the prey evolve mutual weak avoidance and strong alignment. This behaviour appears much like flocking, but the prey tend to disperse over broad areas. If only the sensor  $S_{prey}^2$  is present, misalignment ensues.

If the prey’s obstacle detection sensors are removed, they evolve clumping - strong misalignment and cohesion. If the prey’s predator detection sensors are removed, remarkably, (after a lengthy delay in which they adopt a dispersion strategy) the prey still develop flocking. The minimal sensors needed for realistic flocking to emerge may thus be reduced from five to three - those giving average neighbour velocity, average neighbor position, and average obstacle position.

The predators will stimulate flocking in the prey given only one sensor - the sensor  $S_{pred}^1$  (target prey position) is sufficient.

**Noise** Noise was added to the inputs and outputs of the prey control systems. Whilst this does not capture the expected behaviour of inaccurate or low-grade sensors (which would not produce such consistent output), this does give some perspective on the stability of the flocking observed above when faced with less-than-ideal physiological capabilities.

Adding broadband noise to the prey’s input sensors results in control systems being increasingly disrupted. The prey’s movement becomes agitated. They still, however, evolve to

display some signs of flocking. The effect of sensor  $S_{prey}^1$  (fellow prey position) - through the genomic weight  $c_{prey}^1$  - appears to increase in response to increased noise. This allows the prey to stay close together despite the fluctuations in their senses.

Reducing the SNR below around 0.5 results in unpredictable behaviour - some runs producing rough alignment, others producing other behaviour. Below this region, the weights continue to converge to various figures, but the movement becomes highly erratic and uncoordinated and no apparent coherent behaviours are observed.

Adding noise to the outputs produces similar behaviour. Note the SNR values given here are very approximate, since the signal levels involved exhibit great variation.

**Obstacles** Additional obstacles (20) were introduced within the perimeter boundary after flocking had evolved (Figure 9). Prey agents retained their flocking abilities in the radically altered environment, reducing their mutual distance in response to the altered topology (Figure 10).

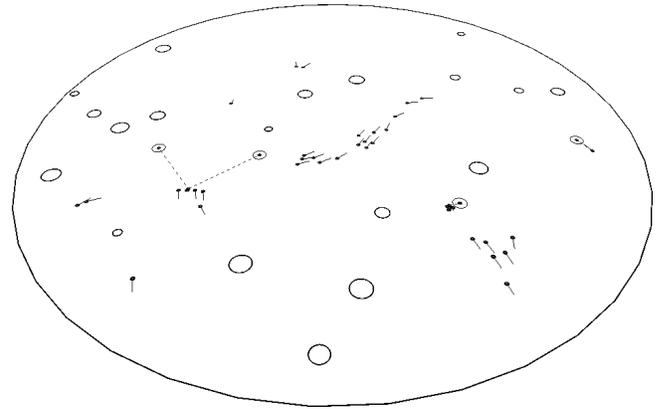


Figure 9: Obstacles added.

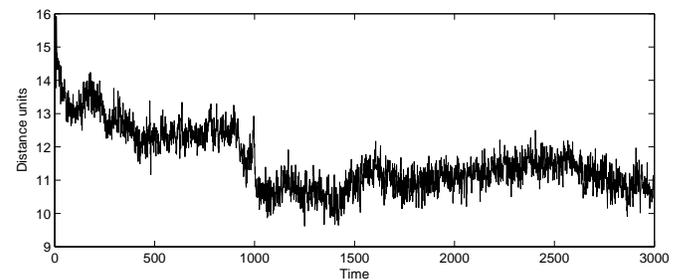


Figure 10: Effect of adding obstacles on inter-prey distance - average of 30 runs, obstacles added at  $t = 1000$

## Discussion

This work adds to the chorus of voices, and evidence, pointing to predation as a major cause of the development of flocking behaviours.

The experiments detailed above demonstrate coevolution of robust flocking strategies in simple vehicles. There has been interest in elaborate neuro-evolutionary flocking models in recent years. It is important that this exploration of complexity be balanced by an understanding of the minimum requirements for such phenomena to exist. The simulation presented here suggests that there is a large amount of order for free, to borrow Stuart Kaufmann's phrase, in an emergent form of flocking behaviour only requiring minimally cognitive agents.

One major question left unanswered is the precise sensory requirements for an agent to be able to perceive the relative positions and velocities of its neighbours. It seems likely that possession of such perceptive abilities (at least to some level of fidelity) is necessary in order for flocking to occur. In animals, flocking is achieved through the use of, e.g. vision, echolocation and lateral line senses. For artificial life purposes, we may add other modalities - as well as methods of communicating the information directly from agent to agent. The present model suggests that, if evolving agents under the influence of predation have the ability to detect this information about their neighbours, then flocking is strongly prejudiced to develop.

### Cognitive Implications

Our model suggests that the level of individual cognitive ability necessary for flocking to develop is very low, given well-chosen sensory and motor abilities. Such minimal models of cognition (as propounded by Beer (2000), Harvey et al. (2005), Duijn et al. (2006)) may prove to be the building blocks for robust theories of higher-level cognition. The cognitive work currently being carried out by inputs, outputs and control systems in our model might be gradually replaced with more realistic versions - whilst investigating the effect on selective pressure towards flocking. The balance being struck here illustrates the blurred boundaries that exist between brains, bodies and environment in the performance of cognitive functions - as described by Clark and Chalmers (1998) - and a careful approach to examining the tradeoff between these domains in the development of such models may prove useful in characterising the development of cognition from simple sensorimotor systems to higher-level forms.

One useful aspect of an evolutionary approach towards questions of group behaviour is in allowing us to analyse how collective cognition might arise from individual-level survival issues. The use of evolutionary techniques holds promise as a method of taming the immense complexity involved in the design and study of advanced cognitive systems. This is especially true in the study of collective cognition, where multiplicity of agents makes scalable design a profoundly important issue. Real-world evolution is capable of producing collective systems with cognitive abilities far beyond our abilities to simulate. While there may be other methods of achieving such aims, evolution should serve (at the least) as an important source of inspiration for future modeling and analysis of complex groups of interacting agents.

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